International Doctorate in Civil and Environmental Engineering

DOCTORAL COURSE

Invariance principles in continuum mechanics

Teacher: Prof. Dr. **Paolo Maria MARIANO**

DICEA, Università degli Studi di Firenze

Email: paolomaria.mariano@unifi.it

Calendar	
05/02/2020, 9,30-11,30 – Aula 106 S. Marta – UniFi	Geometry of large strains
13/02/2020, 9,30-11,30 – Aula 106 S. Marta – UniFi	Classes of changes in observers
14/02/2020, 9,30-11,30 – Aula 106 S. Marta – UniFi	External power and invariance
19/02/2020, 9,30-11,30 – Aula 107 S. Marta – UniFi	Lagrangian mechanics: the Nöther theorem
04/03/2020, 15,00-17,00 – Room to be defined - S. Marta – UniFi	Relative power and invariance: emergence of configurational forces in the bulk
06/03/2020, 15,00-17,00 – Room to be defined - S. Marta – UniFi	The Marsden-Hughes theorem and its extension
11/03/2020, 15,00-17,00 – Room to be defined - S. Marta – UniFi	The dissipative counterpart of Nöther's theorem: covariance of the second law - 1
13/03/2020, 15,00-17,00 – Room to be defined - S. Marta – UniFi	The dissipative counterpart of Nöther's theorem: covariance of the second law - 2
Total	16 hours – 8 credits

Program

Invariance under changes in observers is a key concept in mechanics: the structure of physical laws must be independent of the way we look at phenomena, no matter we deal with solids, fluids, or gases. The same balance equations emerge from requirements of invariance of certain entities, so they are not first principles, although they can be postulated as such. If we consider only the hierarchical structure of continuum mechanics in which kinematics is at ground, followed by the statement of balance laws, irrespective of constitutive prescriptions, which accrue at a later stage, the invariance called first upon is the one of the external power alone (internal power emerging as a derived quantity). When we consider material mutations – namely the occurrence of growing defects – a notion of so-called relative power enters the stage. In both cases, rigid-body type changes in observers are considered. If we enforce the invariance requirement, by calling upon

observers that can deform smoothly one with respect to the other, we need to put on the same level derivation of balance equations and constitutive structures. In conservative setting, the landmark is Nöther's theorem: every transformation leaving invariant the Lagrangian has a correspondent conserved quantity. What can we do when we do refer to dissipative processes? The Marsden-Hughes theorem is a step for answering the question but it does not account for dissipative stresses as in viscous flows. To consider fully dissipative processes, we need to call upon structure invariance of the second law of thermodynamics under changes in observers deforming one with respect to the other – and we call covariance such a requirement. All these foundational questions become operative tools allowing us to construct "safely" models under nonstandard circumstances.
In the present course, we explore these concepts along a path leading to the following results: the basic structures of continuum mechanics emerge all from a unique principle, namely covariance of the second law of thermodynamics.